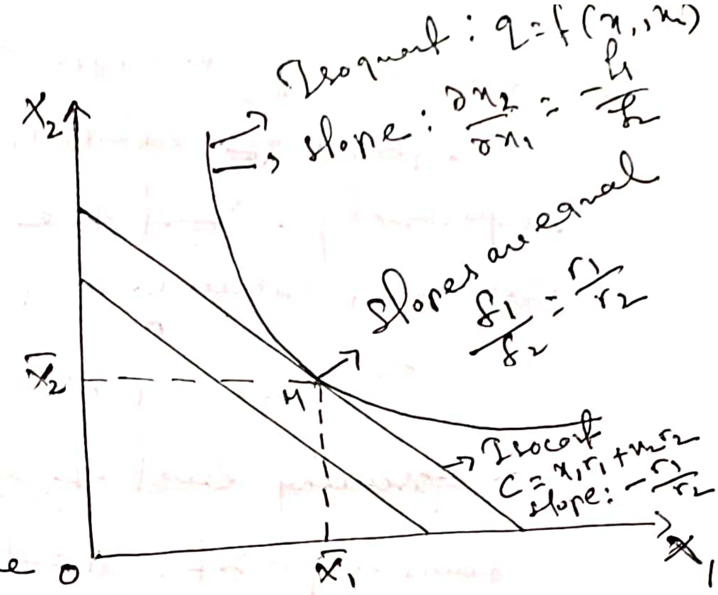
Constrained Profit Maximisation



The point M involves minimum cost to the firm; and also a point of maximum profit to the firm, assuming that there exists perfect competition in output market.

Let us assume that q^0 level of output maximises the profit of the firm which uses x_1 and x_2 amounts of inputs.

Total profit: $\pi = TR - TC$
$= P.q^0 - (x_1 r_1 + x_2 r_2 + b)$ [P is the fixed unit output price]

The firm would maximise subject to the existence of the production function (given technology): $q^0 = f(x_1, x_2)$.

Combining the functions with lagrange multiplier:

$$z = P.q^0 - r_1 x_1 - r_2 x_2 - b + \lambda [q^0 - f(x_1, x_2)]$$

1st order Condition

$$\frac{\partial z}{\partial q^0} = P + \lambda = 0 \quad \text{——— (1)}$$

$$\frac{\partial z}{\partial x_1} = -r_1 - \lambda f_1 = 0 \quad \text{——— (2)}$$

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$$\frac{\partial z}{\partial x_2} = -r_2 - \lambda f_2 = 0 \quad \text{——— (3)}$$

$$\frac{\partial z}{\partial \lambda} = q^0 - f(x_1, x_2) = 0 \quad \text{——— (4)}$$

Equations (2) and (3) give the same condition: $\frac{r_1}{r_2} = \frac{f_1}{f_2}$ which is satisfied only at point M.

Ex 1. A firm's production function is $q = 12 - \frac{1}{LK}(L+K)$. The prices of labour, capital and output are Rs 1, 4 & 9 respectively. Find the maximum profit combination of capital, labour and output.

The cost equation: $C = L + 4K$.

Assuming level of output $= q$ that maximises the firm's profit: $\pi = 9q - L - 4K$

For constrained profit maximisation

$$Z = 9q - L - 4K + \lambda\left[q - 12 + \frac{1}{LK}(L+K)\right]$$

1st order condition: $\frac{\partial Z}{\partial q} = 9 + \lambda = 0$

$$\frac{\partial Z}{\partial L} = -1 - \lambda L^{-2} = 0$$

$$\frac{\partial Z}{\partial K} = -4 - \lambda L^{-2} = 0$$

The above system of equations gives $L = 3$

$K = 3/2$

$q = 11$

and $\pi = 90$.

Ex 2. The demand functions of two competitive commodities are given to be: $x = 11 - 2P_1 - 2P_2$ and $Y = 16 - 2P_1 - 3P_2$

The average cost of production of the commodities are ~~given~~ constants 3 and 1 respectively. Determine prices and quantities that maximise the profit of the monopolist.

Suppose the monopolist in question produces x and y units of two commodities which are sold at P_1 and P_2 prices:

$$TR = P_1 x + P_2 y$$

$$TC = 3x + y$$

$$TP = \pi = TR - TC$$

$$= (P_1 x + P_2 y) - (3x + y)$$

Substituting x and y in terms of P_1 and P_2 from given demand functions:

$$\pi = P_1(11 - 2P_1 - 2P_2) + P_2(16 - 2P_1 - 3P_2)$$
$$- \{3(11 - 2P_1 - 2P_2) + (16 - 2P_1 - 3P_2)\}$$

$$\pi = 19P_1 + 25P_2 - 4P_1P_2 - 2P_1^2 - 3P_2^2 - 49.$$

(i) 1st order conditions:

$$\frac{\partial \pi}{\partial P_1} = 19 - 4P_2 - 4P_1 = 0 \quad \text{------ (1)}$$

$$\frac{\partial \pi}{\partial P_2} = 25 - 4P_1 - 6P_2 = 0 \quad \text{------ (2)}$$

Solving eqn (1) and (2) we get $P_1 = 7/4$ and $P_2 = 3$.

To ensure that π is maximum at $(7/4, 3)$ we apply 2nd order conditions:

(i) 2nd order condition:

$$\frac{\partial^2 \pi}{\partial P_1^2} = -4 < 0$$

$$\frac{\partial^2 \pi}{\partial P_2^2} = -6 < 0$$

Hence, this condition is satisfied for maximum value.

Therefore π is maximum at $(7/4, 3)$

Since $x = 11 - 2P_1 - 2P_2$

$\therefore x = 3/4$ (on substituting values of P_1 and P_2)

Again $Y = 16 - 2P_1 - 3P_2 = 7/2$

$$\pi_{max} = (P_1 x + P_2 y) - (3x + y)$$

$$= \left[\frac{21}{8} + \frac{21}{2}\right] - \left[\frac{9}{2} + \frac{7}{2}\right]$$

$$= \frac{41}{8}$$

$$= 5.13$$ Ans

Hence monopolist should fix the prices at $7/4$ and 3 for two commodities X and Y respectively.

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